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Pneumatic transport

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Certain aspects of the transport of solid particles by a turbulent airstream are discussed, namely: the conveyance of particles in a horizontal pipe, including those carrying an appreciable electrostatic charge; the mechanism of deposition onto a solid wall; and the behaviour of fine particles in a shear flow, such as that in a round jet.

Rough estimates of the effect of the particles on the gaseous turbulence are made, and a primitive physical explanation is offered of the observed velocity-lag and pressure drop associated with the transport of particles in a horizontal pipe, under conditions where the influence of the particles' weight is significant.

Attention is drawn to the difficult problem of dynamically scaling a two-phase flow, and to the different types of interaction between the phases which can occur in a pipe according to its size, the gas velocity through it, and the physical characteristics of the particles.

The paper is an annotated version of a survey presented to the I.U.T.A.M. Symposium on 'Flow of fluid-solid mixtures' held in Cambridge during March 1969.

1. Introduction

When, about a year ago, George Batchelor invited me to present a review paper on Pneumatic Transport to this Symposium, my immediate reaction was to protest that not only was I ignorant of many parts of the subject, but the glancing contact I had made with them merely served to leave me in a state of confusion. To anyone but Professor Batchelor that might have been accepted as a valid and even formidable excuse; to him it was simply an easy challenge to persuasion; at the same time, I can now confess that my resistance was short and flaccid for, in prospect, a year seemed to allow a reasonable time in which to sort out some of the problems that baffled me. Looking back, all I seem to have achieved is a clearer understanding of why I am confused, so that I might the better be able to communicate that confusion to you!

A perplexing feature of the subject is the extensive range of values the various interacting variables may take, the length and velocity scales of the flow, the particle size and density, the direction of the gravitational force on a particle and its ratio to the aerodynamic force, as well as the magnitude of the electrostatic charge a particle might acquire. A skilful and well-informed reviewer would acquaint you with phenomena covering the whole of that range; I shall attempt to comment only on a few, what appear to me to be, salient features. To that end, I shall first concentrate on the transport of heavy particles through a *horizontal* pipe, since the asymmetry of the gravitational force leads to more complicated and possibly interesting effects on the flow than it does when the pipe is vertical. Then, I shall turn to the behaviour of rather finer particles, which appear to be susceptible to quite significant electrostatic forces. Next, I shall describe certain aspects of the problem of the deposition of particles in the 0·1 to 100 μ size range on to a solid wall, and illustrate some of the consequences of the process by reference to flow in the human lung. Finally, I propose to consider the interaction between small dust particles and a shear flow, in particular, the flow in a jet.

The two features all these flows possess in common are a turbulent motion of the gaseous phase, and an appreciable relative velocity between the solid particles and the gas, at least in some part of the flow field; and those, rightly or wrongly, I have taken as sufficient to define a pneumatic transport.

The fact that the fluid phase is gaseous rather than liquid implies that in considering the fluid mechanical forces exerted on particles we may, in general, leave out of account the Magnus force associated with spin and the lift force arising from the mean shear in the fluid, except possibly very close to a solid boundary, since both, compared with the weight of the particle, are of order of magnitude ρ_P/ρ, if the particle is large (where ρ_P is the density of the particulate material, and ρ the density of the fluid). I mention this now, although I shall make another passing reference to it later, because it seems to be one of the traditions in the study of particle-laden flows to invoke such forces for the explanation of a vast range of phenomena.

2. Flow in a horizontal pipe

As a preliminary, it would be appropriate to describe the broad patterns of behaviour of the solid phase as revealed by experiments on the two-phase flow in a horizontal pipe. Following the observations of Richardson & McLeman (1960), Farbar (1949), Wood & Bailey (1939) and others, it appears that for light to moderate solid loadings (i.e. for ratios of the mass fluxes of particles to air of 10 : 1 and less) the sequence of events following a change in the mean airspeed is as indicated in figure 1, curve I, where pressure drop is plotted against air speed. Here, and throughout this talk, 'pressure drop' refers to the fall in pressure over that section of the pipeline in which the particle velocity at any radius is, on the average, independent of axial position.

At high airspeeds, the particles appear to be in suspension and are distributed more or less uniformly across the entire pipe. Then, as the airspeed is reduced, there is a tendency for particles to congregate in the lower half of the pipe or, in the case of particles of non-uniform size, for the larger ones to travel in the lower part, whilst the smaller ones occupy the upper and faster-moving layers of the stream. It also appears that the particles in the lower part of the pipe may assemble into regions of high concentration interspersed with comparatively dilute regions. Such conditions seem to be realized in the neighbourhood of the minimum in the pressure curve. With further reduction in velocity, deposition occurs, in some cases uniformly and accompanied by saltation, and in others in preferred areas of the pipe wall leading to dune formation. When the velocity is reduced still further, deposition continues until the pipe is completely blocked. Before transport ceases, agglomerates of particles almost filling the pipe may be forced along the wall in the form of sliding dunes, or the dunes may be almost stationary and suffer partial erosion. During this process the pressure drop is large.

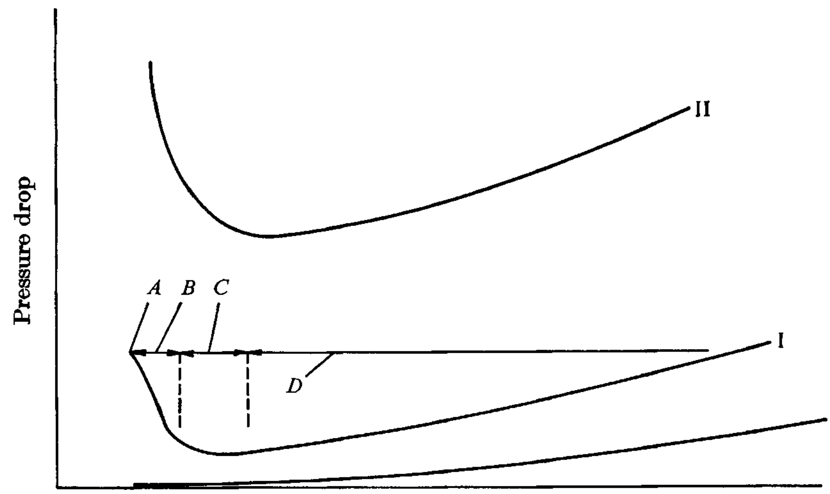

FIGURE 1. Rough description of observed patterns of particle behaviour in an horizontal pipe. — (unmarked), particle-free gas; I, light solid loading; II, heavy solid loading. *A*, blocking; transport ceases. *B*, deposition; dune formation; saltation. *C*, concentration in lower part of pipe. *D*, suspension.

Curve II refers to high solid loadings. The pattern of events is similar to that for a lightly loaded flow, except that dune formation appears to be more typical at low airspeeds than uniform deposition, and saltation does not seem to occur.

According to a view of the saltation mechanism that I described in 1964, the suppression of saltation in a highly concentrated particle flow can be explained by the fact that the shear stress borne by the air falls to too small a value at the bottom of the pipe for the particles lying there to remain mobile and therefore to be ready to participate in the motion. Such a fall in shear stress is required, in a statistically steady flow, to balance the fluid thrust on the particles developed as a result of the lag between their velocity and that of the fluid.

3. Classification of flow-particle interaction in horizontal pipes

Confining attention to light or moderate solid loadings, the behaviour exhibited in figure 1 is consistent with a more general classification of the interaction between the particles and fluid which I suggest can be made as follows.

Starting with very fine particles,† and a turbulent airstream, we might imagine that at high enough airspeeds the particles very nearly follow the energetic component of the motion of the gas, how very nearly I shall comment on later. As the particle size increases, so that the particles' relaxation time‡ t^* becomes comparable with the characteristic time scale for the energy-containing eddies of the turbulence, the response of the particles to those eddies becomes imperfect. If the scale of the mean motion is the radius of the pipe a (so that, guided by the experiments on homogeneous flows of Townsend (1956), the scale of the energetic eddies is of order $10^{-1}a$) and the friction velocity at the pipe wall is u_τ, the condition for disengagement from the energy-containing eddies is

$$t^* \sim 10^{-1}a/u_\tau. \tag{1}$$

Since I am concerned only with orders of magnitude, it is sufficient to calculate t^* according to the Stokes' law of resistance:

$$t^* = \frac{1}{18}\frac{\rho_P}{\rho}\frac{d^2}{\nu}. \tag{2}$$

In (2), ρ_P is the density of the particulate material, and d is the particle diameter, here assumed to be uniform. Hence the condition for imperfect response to the energy-containing eddies becomes

$$\frac{5}{9}\frac{\rho_P}{\rho}\frac{u_\tau d}{\nu}\frac{d}{a} \sim 1. \tag{3}$$

The next main event, which occurs with a further increase in particle size, is disengagement from the large eddies, whose scale is comparable with the pipe radius. The condition for that is

$$t^* \sim a/u_\tau, \tag{4}$$

or

$$\frac{1}{18}\frac{\rho_P}{\rho}\frac{u_\tau d}{\nu}\frac{d}{a} \sim 1. \tag{5}$$

So far, the classification involves only the ratio of the diameters of particle and pipe, together with the Reynolds number $u_\tau d/\nu$. But now we have to include the effect of gravity.

Broadly speaking, gravity has an influence on the two-phase flow when

$$v_0 \sim u_\tau, \tag{6}$$

where v_0 is the terminal velocity of a particle in a still fluid. More particularly, it may induce saltation if, as I suggested in my 1964 paper,

$$O(1) > \frac{\rho u_\tau^2}{\rho_P g d} > O(10^{-2}), \tag{7}$$

or it can lead to copious deposition if

$$\frac{\rho u_\tau^2}{\rho_P g d} < O(10^{-2}). \tag{8}$$

† Here and subsequently, when I refer to particle size, I associate it with a density of the material of the order of a thousand times that of air.

‡ t^* is the time taken by a particle to adjust to a change in environment.

The term 'copious' is used to distinguish the intensity of deposition from that which must occur in order to enable a saltation to be maintained. The introduction of gravitational effects thus involves the additional parameter $\rho u_\tau^2/(\rho_P g d)$, which is a kind of Froude number.

Turning back to very fine particles, typically submicronic in diameter, Brownian diffusion significantly affects their motion in the vicinity of a wall, if that diffusivity D is comparable with the eddy diffusivity within the viscous sublayer. The condition for this to be so is

$$\frac{u_\tau d}{\nu} \sim 10\left(\frac{D}{\nu}\right)^{\frac{1}{3}}. \tag{9}$$

(See note (i).)

One could, without difficulty, extend and refine the classification by introducing more phenomena and therefore more independent parameters.

For example, having assumed the system to be dilute we may ignore the drain on turbulent energy provided by the work done in sustaining the particles against gravity.† For that rate of work to be negligible in comparison with the input from the mean flow we require,

$$-\frac{g}{\rho}\frac{d\sigma_P}{dz}\bigg/\left(\frac{dU}{dz}\right)^2 \ll 1, \tag{10}$$

where z is measured vertically upwards, and σ_P is the mass concentration of solids.

As I mentioned before, we may also ignore lift forces on particles due to their spin and to shear in the fluid. The condition that such forces are small compared with the weight of a particle can be shown to be

$$\frac{\rho}{\rho_P}\frac{u_\tau^2}{gy} \ll 1 \tag{11}$$

provided that $u_\tau d/\nu \gg 1$ (see note (ii)), and that the particles are so massive that their relaxation time is greater than the lifetime of an energetic eddy. y is the distance measured from a solid wall. Since ρ/ρ_P for pneumatic transport is $O(10^{-3})$, (11) is almost invariably satisfied (but not so in hydraulic transport).

Based on the foregoing classification, the domains of flow-particle interaction are shown in figure 2, for pipes of 5 cm and 50 cm diameter, representative of the laboratory scale and full-scale. I have just a few lugubrious comments to make on this figure. In the first place, we probably have to accept that hopes for developing the kind of detailed theory of particle motion, like that put forward by Tchen (1947), amended by Corrsin & Lumley (1956) and reviewed by Hinze (1959), diminish as we go to the right along the diameter scale. In that respect, it has to be recognized that the practical systems of pneumatic transport of interest to the chemical engineer fall into the most complex of all the regions in the figure: that in which gravity is important, and in which response to even the largest scale components of the turbulence is imperfect. A further complication in a practical system arises from the non-uniformity of particle size, so that more

† Collisions between particles may also be ignored.

than one domain may be occupied for any given gas velocity. Finally, we notice that the positions of the boundaries are dependent upon the linear scale of the flow and, since that dependence involves both Reynolds number and Froude number, the problem of exact dynamical scaling in a laboratory is a very difficult one indeed. As a matter of fact, this question of scaling is one which I rarely find discussed in the experimental literature on pneumatic transport.

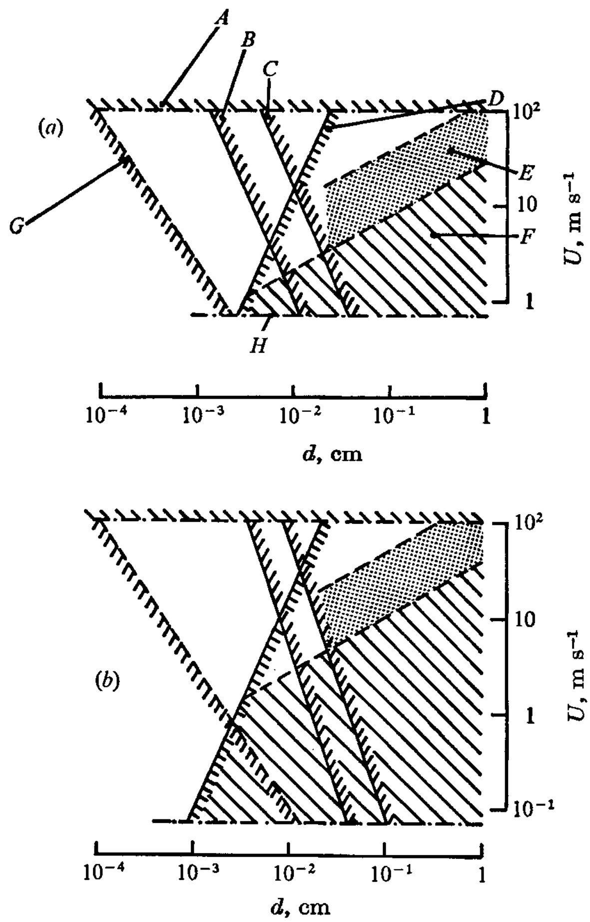

FIGURE 2. Domains of particle-flow interaction in horizontal pipes: (*a*) 5 cm diameter. *A*, compressibility of the gas significant. *B*, imperfect response to energetic eddies. *C*, imperfect response to large eddies. *D*, gravity significant. *E*, saltation. *F*, copious deposition. *G*, Brownian diffusion significant close to the wall. *H*, limit of turbulent flow in a particle-free gas. (*b*) 50 cm diameter.

Whilst the conclusions I have drawn from the figure might suggest a certain despair of the problem, fortunately many chemical engineers have taken a more robust attitude towards it, in some cases with a happy disregard of its complexities. According to a census taken in 1964, over 20,000 measurements of the pressure drop in two-phase flow systems had been made, half of them during the previous 5 years. Since then, another 5 years have passed! The authors of that census, Dukler, Wicks & Cleveland (1964), go on to say 'this continued accumulation of data demonstrates that there is not yet even a phenomenological understanding of this type of flow'. Perhaps, I should have read that passage to George Batchelor a year ago!

4. The experiments of Richardson & McLeman on transport in a horizontal pipe ($d \sim 10^{-1}$ cm)

However, I do not believe the situation to be quite so gloomy, and I now want to pick out what in my view is one of the outstanding experimental contributions to the subject of pneumatic transport in horizontal pipes made in recent years:

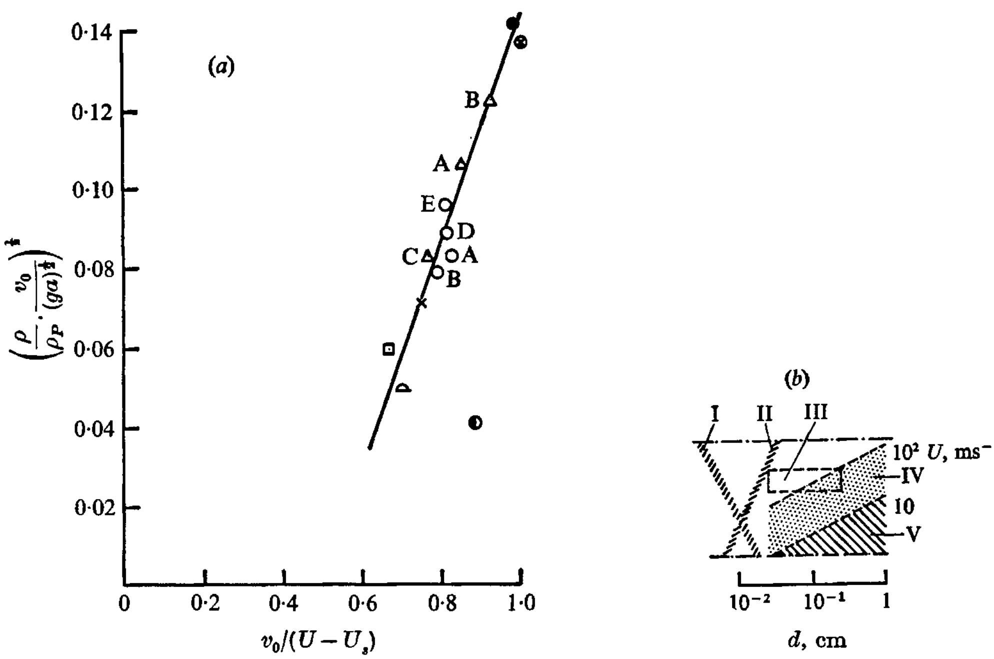

FIGURE 3. Analysis by Richardson & McLeman of the velocity of particles relative to the air in the flow through a 1 in. diameter horizontal brass pipe. (*a*) ●, radish; ⊗, rape; △, perspex; ○, coal; ×, polystyrene; □, aluminium; ⌓, lead; ◐, brass. (*b*) I, imperfect response to large eddies. II, gravity significant. III, region of experiments. IV, saltation. V, deposition.

that by Richardson & McLeman (1960). They measured the relative velocity of the two phases and the pressure drop for a variety of particles, and derived remarkable empirical expressions for them.

Figure 3 shows such an empirical relation for the relative velocity. It is a slightly bowdlerized version of the original, since I have made the ordinate non-dimensional. However, the broad conclusion to be reached from this figure (which, by the way, applies to particles possessing a rather compact size distribution) is that the relative velocity is comparable with the terminal velocity of the particles, and is not a very powerful function of the ordinate variable.

To interpret such a relation physically, I must first draw attention to the inset diagram at the bottom right of the figure which shows the domain of particle-flow interaction occupied by the experiments. It is firmly embedded in the region where gravity is significant, and where the response of a particle to even the largest scale features of the turbulence is imperfect. Moreover, it straddles the saltation boundary. Now, for reasons which will be given presently, that does not imply that saltation will necessarily have occured, but it does suggest that we might appeal to certain properties of the saltation process in guiding our analysis of the particle behaviour.

One such property, first recognized by Bagnold (1941) is that, after impact, particles leave the wall in a predominantly radial direction. In that case, particles travelling inwards towards the pipe axis acquire an axial velocity given approximately by

$$U - U_s = m\{\pi C_D \rho d^2 t/8 + m/U\}^{-1}, \tag{12}$$

where U is the airspeed, taken to be uniform, m is the mass of a particle, and t the time of flight. The drag on a particle is assumed to vary quadratically with its velocity relative to the fluid. (See note (iii).)

The second property of a saltating flow to emerge from my 1964 analysis is, that the time of upward flight is of order v_0/g (i.e. comparable with the relaxation time). A glance at the table 1 reveals that the time of flight for the solids listed there is of the order of one second which, at airspeeds of up to about 30 m s^{-1} appropriate to the experiments, would lead to trajectories with apogee heights of the order of 1 m, if the particles were able to perform a pure saltation. Evidently they could not do so in a pipe just 1 in. in diameter. Instead, we are led to hypothesize a wall-constrained saltation† in which the particles travel across the pipe (but not, of course, in diametral planes) with a radial component of velocity V_s, reduced in comparison with that in an unbounded saltation, and so adjusted that a single transverse excursion through a distance comparable with the pipe radius is completed in a time t^*. Accordingly, t in the above expression for $U - U_s$ is, in order of magnitude, v_0/g; and, since $\pi C_D \rho d^2/(8m) = g/v_0^2$, it follows that

$$U - U_s \sim v_0, \tag{13}$$

neglecting the small term v_0/U. U_s may now be interpreted as the average axial component of velocity of a particle. (See note (iv).)

It may be noted that the radial velocity V_s suggested by the above argument is of order ga/v_0, and is consistent with the particle motion being generated initially by the turbulence. Thus, if the velocity with which a particle approaches the wall, and subsequently rebounds from it, is the result of an interaction between the particle and a turbulent eddy of characteristic velocity u_τ and length scale a, that velocity is approximately

$$u_\tau\{1 + (t^*/t_e)^2\}^{-\frac{1}{2}} \approx u_\tau t_e/t^*,$$

where t_e is the eddy lifetime, typically a/u_τ. Hence, we recover the expression

$$V_s \sim a/t^* \sim ga/v_0.$$

(See note (v).)

† We must again be cautious about the effect of scale, because such an hypothesis might be unnecessary if the pipe were of one metre diameter or more.

Having deduced in form the empirical relation of Richardson & McLeman for the relative velocity through such a simple argument, it is tempting to see whether the relation those authors derived for pressure drop can also be interpreted physically. That relation is shown in figure 4. There, I have not tampered at all with the original. It is an extraordinary achievement in the art of correlation.

Particle	Mean dia. cm	Terminal vel. m s^{-1}
Radish seed	0·25	6·5
Rape seed	0·19	5·9
Perspex A	0·15	3·7
Perspex B	0·37	5·0
Perspex C	0·075	2·4
Coal A	0·075	2·8
Coal B	0·063	2·4
Coal D	0·10	3·3
Coal E	0·20	3·7
Polystyrene	0·035	1·6
Aluminium	0·022	3·0
Lead	0·030	8·2
Brass	0·037	4·1

TABLE 1

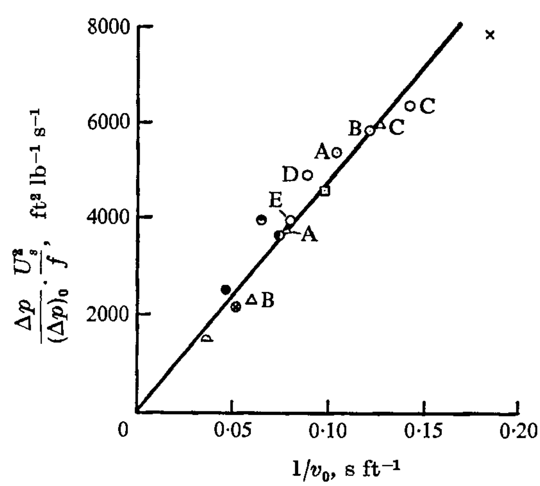

FIGURE 4. The Richardson & McLeman correlation of pressure drop in the flow through a 1 in. diameter horizontal brass pipe. ×, polystyrene; △, perspex; ○, coal; □, aluminium; ◓, sand; ◐, brass; ●, radish; ⊗, rape; ⌓, lead.

The line through the experimental points corresponds, in an f.p.s. system of units, to

$$\frac{\Delta p}{(\Delta p)_0}\,\frac{U_s^2 v_0}{f} = 4{\cdot}5 \times 10^4\ \mathrm{ft^3\,lb^{-1}\,s^{-2}}.$$

In interpreting such an expression, we have to be careful not to use the previous relation (13) to work out the thrust on the particles, since it is essentially an average across the pipe, and is not accurate enough to provide a measure of $(U-U_s)^2$ on which the total thrust depends. Instead, let us approach the problem by way of the momentum transfer to the wall through particle impact.

Again, appealing to saltation, we assume that a particle on impact loses an appreciable fraction of its component of axial momentum. Accordingly, over a length dx of pipe the pressure drop is given by

$$a^2 \Delta p \sim mnUaV_s dx, \tag{14}$$

where n is the number concentration of particles. Here I imagine an incoming particle to be travelling near the wall with axial velocity U (or nearly so) and to be driven towards the wall with velocity V_s by a turbulent eddy. But we have already shown that

$$V_s \sim ga/v_0. \tag{15}$$

Recognizing that

$$mna^2 U_s \sim f, \tag{16}$$

the mass flux of solid, we then obtain

$$\Delta p \sim fUg(a^2 U_s v_0)^{-1} dx; \tag{17}$$

since, for a particle free flow

$$(\Delta p)_0 \sim \rho U^2 dx/a, \tag{18}$$

it follows that

$$\Delta p/(\Delta p)_0 \sim fg(\rho a U U_s v_0)^{-1}, \tag{19}$$

which differs only by the factor U_s/U from the Richardson–McLeman expression.† But $U_s/U \approx 1 - v_0/U$, which is not very different from unity.

The dependence of the pressure drop ratio on $1/a$ is consistent with the behaviour of Segler's (1951) measurements on the flow of wheat in pipes of different radius, as noted by Richardson & McLeman. It is shown in figure 5.

5. Electrostatic effects ($d \sim 10^{-3}$ cm $- 10^{-2}$ cm)

I have discussed this interpretation of Richardson & McLeman's results at some length to show that, after all, there may be a ray of hope for a physical understanding of pneumatic transport in pipes under conditions of concern to the chemical engineer: significant gravitational influence and imperfect response to turbulent velocity fluctuations. Having said that, I must immediately draw attention to a further complication which arises from the existence of electrostatic charges on certain types of particle. The presence of such charges had been recognized as long ago as 1900 and measurements of the electric field caused by them were made in dust storms in the Sahara. Later experiments, notably by Whitman (1926), on particles in the sub μ to 100 μ diameter range travelling through tubes, revealed, for certain particulate materials, such as quartz, charges as large as 10^{-4} C/kg of substance. Later still, Kunkel (1950), observed that the average charge per particle increased with size rather more slowly than

† In fact, replacing the π's in the above analysis, and recalling that a factor $O(10^{-3})$ should appear on the right of (18), an estimate of the value of the Richardson–McLeman constant in the relation (19) for $\Delta p/(\Delta p)_0$ is $O(10^5)$ in an f.p.s. system of units and for a pipe diameter of 1 in. They gave it as $4{\cdot}5 \times 10^4$.

the square of the diameter or surface area, so that the charge per kilogram may be expected to decrease with increasing diameter.†

For particles of sand, granite and carborundum in the 1 mm range of diameters, it had been noted by Clark, Charles, Richardson & Newitt (1952) in their experiments on pneumatic transport in a pipe that, if a particular material is conveyed for a long period, the pressure difference required to maintain the transport progressively increases and can reach a value some 10 times greater than the initial one. The observations were carried further by Richardson & McLeman (1960), who found that not only the pressure drop increased, but so did the

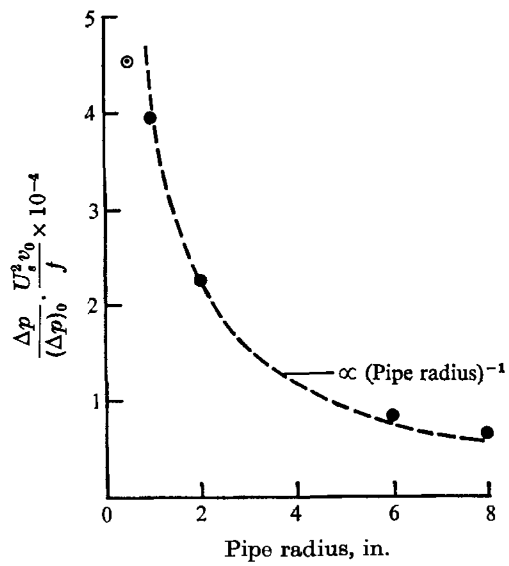

FIGURE 5. Effect of pipe radius on the pressure drop due to the transport of wheat in a horizontal pipe. ●, Segler (wheat grains) analyzed by Richardson & McLeman; ⊙, Richardson & McLeman. U_s ft s^{-1}, v_0 ft s^{-1}, f lb s^{-1}.

relative velocity between the particles and fluid. They suggested that the phenomenon might be explained by the acquisition of charge of opposite sign by the main body of the particles, and by the dust layer left on the wall of the pipe through attrition of the particle material. Thus, the violence of particle impact with the wall is increased by the electrostatic attraction, which would account for an increase in both relative velocity and pressure drop.

However, if we now turn our attention to smaller particles, in the 10 to 100 μ diameter range, and therefore possessing larger charge/weight ratios, quite different effects attributable to electrostatic charging have been observed. These involve a *reduction* in pressure drop accompanying the presence of solid particles.

At this point, I should emphasize that I am referring to moderately dilute systems, and not to those of such high concentration as to render the two-phase

† Descriptions of the effects of electrostatic charge, together with extensive bibliographies relating to this and other aspects of particle behaviour, can be found in the books by Dalla Valle (1948), Fuchs (1964), Green & Lane (1964) and Soo (1967).

fluid non-Newtonian. These latter conditions have been investigated extensively by Thomas (1963).

For the more dilute systems, Soo (1965) examined the pneumatic flow of magnesium oxide particles; his results on pressure drop, taken from his book (1967), are shown in figure 6. Seeing such data in isolation, one might be inclined to wonder whether the behaviour could be associated with the method of pressure measurement rather than the flow itself, but such doubts are removed by appeal to another set of measurements by Tien & Quan (1962), this time on heat transfer. Their observations are shown in figure 7 (also taken from Soo's book), and again a pronounced reduction in the rate of transport to the wall is evident.

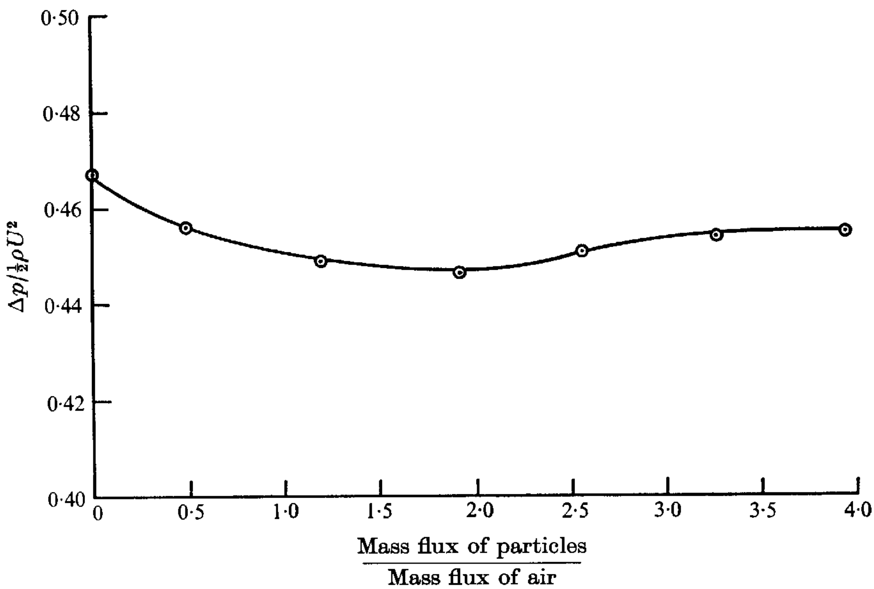

FIGURE 6. Measurements by Soo of the pressure drop accompanying the transport of 35 μm MgO particles in air through a 12·5 cm brass pipe. Mean airspeed 42·6 m s^{-1}.

As several authors have observed, presumably the explanation lies in a reduction in the turbulence of the gaseous phase due to the presence of the particles. In fact, a reduction in turbulence has also been surmised by experimenters working with *larger* sizes of *uncharged* particles, e.g. Clark, Charles, Richardson & Newitt (1952). There, one might imagine that the particles, in lagging behind the mean motion of the gas, behave like a cascade of gauze screens with respect to the flow relative to them, in the sense of slowing-down the faster moving parts of the gas and speeding-up the more slowly moving parts.†

† What those authors deduced was a reduction in the gaseous *skin friction* due to the presence of the particles. In that case, a simpler alternative explanation is possible. In a statistically steady flow, an increasing proportion of the shear stress developed in the gas is transferred to the particles, in the form of a momentum flux, as the wall is approached. Accordingly, a reliable method of distinguishing the contributions to skin friction from the gas and from the particles might well indicate a reduction of the former in the presence of the latter, even if the turbulent structure in the central parts of the flow were unaffected.

In the case of the smaller particles, appropriate to the measurements displayed in figures 6 and 7, I believe that one must seek a rather different physical explanation and invoke the presence of electrostatic charge. That such a charge exists, under the conditions of Soo's experiment, has been demonstrated by him and co-workers (1964); but we can infer its presence indirectly from their measurements of the particle concentration distribution (figure 8).

In the absence of a radial force field, such small particles would be distributed uniformly across the pipe, but the upper diagram indicates a pronounced increase in concentration towards the wall. That increase is consistent with the

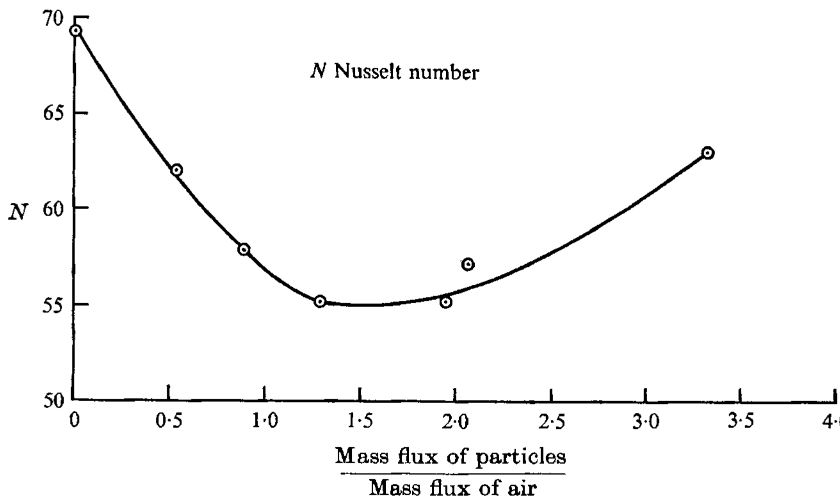

FIGURE 7. Measurements by Tien & Quan of the heat transfer accompanying the transport of 30 μm glass particles in air through a pipe.

presence of an electrostatic field tending to drive particles towards the wall, compensated by an opposing turbulent transport towards the axis, so that, on the average, there is no net deposition on the wall.

The lower part of figure 8 shows the corresponding mean velocity profiles for the particles and the gas. They again are consistent with a comparatively slow drift of particles towards the wall under the action of the electrostatic field (such particles possessing approximately the mean axial gas velocity), balanced by a more violent eruption of particles predominantly radially inwards, due to the turbulent eddies near the wall. An estimate of the radial distance travelled by such particles, under the conditions of Soo's (1965) experiment, gives it to be of the order of magnitude of 1 cm, which is comparable with the distance from the wall at which the maximum departure of the mean axial velocity of the particles from that of the gas is observed; and that is what one would expect. Referring again to the upper diagram of figure 8, we may suppose that particles tend to congregate, awaiting ejection by an eddy, at such a distance from the wall that their relaxation time is comparable with the eddy lifetime there; for, if they amassed closer to the wall, the mechanism for projecting them back towards the pipe axis would be enfeebled, since the eddy lifetime decreases as the wall is approached. In that case, the mean velocity gradient in the region of maximum concentration is of order $1/t^*$, and the rate of energy input to unit volume of the turbulence $\rho u_\tau^2/t^*$.

Due to the electrostatic force field, which I assume arises from charges only on the particles themselves, the particles can be thought of as being acted on by an apparent gravity g^*, under which they tend to drift towards the wall with velocity v^*. In fact,

$$v^* = g^* t^*. \tag{20}$$

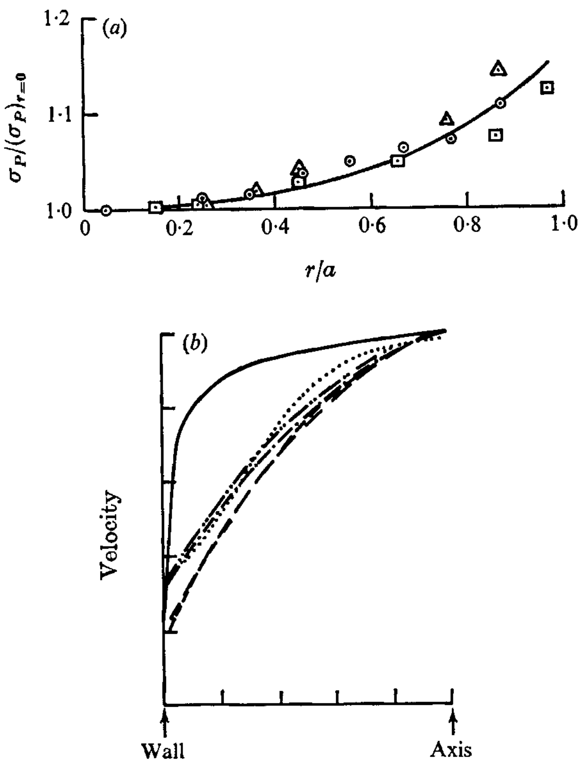

FIGURE 8. (*a*) Measurements by Soo of the mass concentration σ_P of 50 μm glass particles in pneumatic transport through a 12·5 cm brass pipe. Approximate mass flux ratios, particles/air, β: ○, 5; □, 8; △, 10. (*b*) The mean velocity distributions of glass and MgO particles. —, particle-free flow. ......, MgO; $\beta = 1{\cdot}0$. —··—, MgO; $\beta = 0{\cdot}5$. —·—, MgO, $\beta = 0{\cdot}25$. — — —, glass; $\beta = 8{\cdot}0$. - - - -, glass; $\beta = 5$.

(See note (vi).) If σ_P is the mass concentration of particles near the wall, the rate at which work must be done by the turbulence to maintain zero net rate of particle deposition, on the average, is

$$g^* v^* \sigma_P \tag{21}$$

per unit volume of fluid.

The ratio of this rate of working to that of energy transfer from the mean gas flow to the turbulence is

$$(\sigma_P/\rho)(v^*/u_\tau)^2; \tag{22}$$

which is a (L.F.) Richardson number. From experiments on single phase density-stratified flows, such as those of Ellison & Turner (1959) and Webster (1962), it is known that the turbulence is affected appreciably when the Richardson number exceeds about 0·1.

For the magnesium oxide particles to which figure 8 showing the pressure drop referred, I find that for ratios of particle mass flux to that of the gas of the order of unity, Richardson numbers of 0·1 are achieved when the charge/mass ratio is of the order of 10^{-6} C/kg. In fact, that is just the order of magnitude found by Soo *et al.* (1964) in their measurement of electrostatic charge.

6. The influence of particles on the turbulence

It appears then that larger, uncharged particles, such as those studied by Clark, Charles, Richardson & Newitt (1952), possibly affect the turbulence over most of the cross-section of the pipe, whereas smaller particles carrying an electric charge have their main influence in regions of comparatively high concentration and large electrostatic field near a wall. In both cases, the particles are small compared with the radius of the pipe, so that the wakes shed from them make no energetic contribution to the turbulence. This is in contrast to what I imagine happens in a pure *saltating* flow, where the particles must be considered massive compared with the aerodynamic forces acting on them. In those circumstances, I have argued (1964) that the turbulence level is directly controlled by the particle motions, indeed it is *sustained* by them, and that the scale of the eddies is comparable with the vertical scale of a particle trajectory. In particular, if the saltation is occurring in the atmosphere, the saltating particles act on the flow away from the ground in the manner of an aerodynamic roughness, an hypothesis supported by the measurement of wind profile made by Bagnold (1941), Chepil (1945) and Zingg (1953), as shown in figure 3 of my 1964 paper.

Here, then, is a situation in which the particle motion feeds energy to the turbulence, at the expense, of course, of the mean flow. That is because the particles are so massive that they dominate the smaller-scale motion of the gas.

For finer particles, I shall give my view, for what it is worth, on the question of interaction between the particles and the gaseous turbulence. Here, I start from the premise that the particles are uncharged, and that the energy input to the turbulence is afforded entirely by the mean motion of the gas, so that the particles are, so to speak, provided with a pre-formed field of turbulence with which to interact.

The quantity controlling that interaction is surely the relaxation time of a particle. First, consider particles so fine that

$$t^* \ll t_e,$$

where t_e is the characteristic time of an energy-containing eddy in the gas. Then, relative to that eddy, the particle will possess a motion only during a time comparable with t^*. In that period, the particle is acted on by a force

$$Ku,$$

where u is the turbulent velocity fluctuation in root mean square and, for simplicity, I am assuming the relative flow to be of the Stokes type, so that

$$K = 3\pi\mu d,$$

and the total work done by the eddy is

$$Ku^2t^*.$$

If there are n particles per unit volume, the average rate of work during an eddy lifetime is

$$nKu^2t^*/t_e.$$

But $t^* = m/K$. Hence, the average rate of working turns out to be

$$\sigma_P u^2/t_e \sim \sigma_P u^3/l \tag{23}$$

where l is the scale of the turbulence.† Consequently, the rate of turbulent energy dissipation compared with that in the particle free fluid is increased in the ratio $(1+\sigma_P/\rho)$.

If the energy production and energy dissipation proceed at comparable rates

$$\rho ul(dU/dy)^2 \sim (\rho+\sigma_P)u^3/l, \tag{24}$$

in which ul may be interpreted as an eddy viscosity. For a given mean velocity profile, we might plausibly suppose that the turbulent scale l is unaffected by the presence of the particles, if their concentration is sufficiently dilute to satisfy the condition (10). In that case, u is decreased by the particles in the ratio

$$(1+\sigma_P/\rho)^{-\frac{1}{2}}.$$

Accordingly, the eddy viscosity may be expected to decrease in the same ratio.

For more massive particles, such that $t^* \gtrsim t_e$, a similar argument, allowing for the fact that a particle responds partially to the turbulent velocity in the time t_e, shows that

$$u(\sigma_P)/u(\sigma_P = 0) \sim \{1+(\sigma_P/\rho)(t_e/t^*)\}^{-\frac{1}{2}}. \tag{25}$$

Finally, when $t^* \gg t_e$, the particles are almost insensible of the turbulent fluctuations. Instead, they possess on the average some mean velocity relative to the fluid. Accordingly, as I suggested before, they may be regarded statistically as fixed centres of resistance, with the fluid and its turbulent structure flowing past them, so that we might think of their action on the turbulence as a distortional one, similar to the action of a gauze screen.

7. Particle deposition ($d \sim 10^{-5}$ cm $- 10^{-2}$ cm)

In the description of the behaviour of particles in an electrostatic field given in §5, I referred to conditions near the wall. I should now like to examine those conditions more carefully and, in particular, to see how they might control the process of particle deposition. For this purpose, I shall assume electrostatic effects to be absent, and concentrate on fine particles of diameter $10^2\,\mu$ and less. The surface under consideration is supposed to be smooth, and may form the wall of a pipe or channel or even a ground exposed to atmospheric wind.

In general, the transport of fine solid particles from a turbulent gas stream to an adjoining surface is provided by turbulent diffusion, except very close to the

† The same result is obtained if the assumption of a Stokes flow about the particle is relaxed. Thus, if the drag on the particle is taken to be K_2u^2, the average rate of work during an eddy lifetime is $nK_2u^3t^*/t_e$, but now $t^* \sim m/(K_2u)$, and we again derive $\sigma_P u^3/l$ for that rate of work.

surface in the viscous sublayer, where the turbulent diffusivity vanishes as the wall is approached. *Very* small particles, of diameter less than about 1 μ, are able to traverse the viscous sublayer through the action of their Brownian motion. For larger particles, whose Brownian diffusivity is small (it varies approximately inversely as the diameter), some more powerful mechanism must be called into play to enable the particles to reach the wall. Friedlander & Johnstone (1957) and myself (1960) independently suggested that particles are projected towards the wall by eddies near the edge of the viscous sublayer. On reflection, I find that such a theory possesses an inherent inconsistency. It is simply that in responding vigorously to turbulence in the outer part of the viscous sublayer, the relaxation time of the particle is supposed to be small compared with the characteristic time of an eddy, yet, to traverse almost unimpeded the remainder of the viscous sublayer, the relaxation time has to be large compared with that same characteristic eddy time.

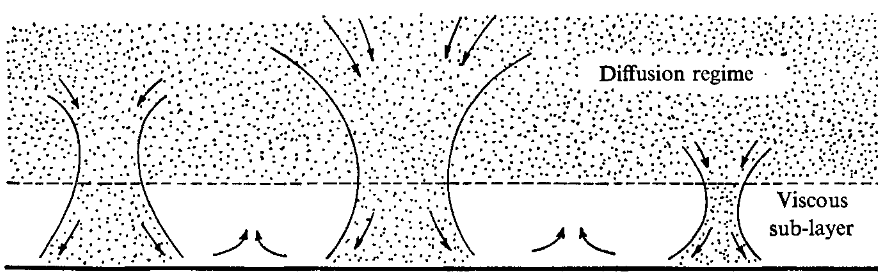

FIGURE 9. Model of the eddy system responsible for the convection of particles towards a wall. The direction of the main flow is normal to the paper.

As an alternative, I now propose that particles are *convected* to the wall from the region of energetic turbulent motion outside the viscous sublayer by the occasional large eddy that encroaches on it, as suggested in figure 9.† Here, the main flow is supposed to be streaming into or out of the paper, and the eddies that transport material towards the wall can be thought of as occurring in response to the sporadic violent eruption from the viscous sublayer, as observed by Kline *et al.* (1967).

Away from the wall, the particles migrate under the action of turbulent diffusion, and the so-called 'diffusion regime' is distinguished from the 'convection regime' adjacent to the wall by the condition that, within the former, the relaxation time of a particle is short compared with the lifetime of a turbulent eddy, whereas within the convection regime it is not.

In fact, the boundary between the two regimes can be blurred by appealing to Laufer's (1954) pipe measurements, which are consistent with the turbulent velocity component normal to the wall, in root mean square, increasing with distance from the wall like its $\frac{1}{3}$ power.

It can then be shown that the number of particles deposited on unit area of the wall in unit time is given by

$$N = Ru_\tau n_0 \sigma^2. \tag{26}$$

† A full description of this work is intended to be published shortly.

n_0 is the concentration of particles away from the viscous sublayer, and σ is a quantity proportional to the ratio of the particle relaxation time to the characteristic eddy time in the flow near the wall. It is defined by

$$\sigma = u_\tau^2 t^* / \nu. \tag{27}$$

R is a constant estimated in order of magnitude to be 10^{-4}.

The rather simple expression for N that emerges from this argument is compared in figure 10 with the measurements of Friedlander & Johnstone (1957) (open circles) and of Wells & Chamberlain (1967) (black dots). The slope of the line is what is predicted, and corresponds in position to a value of the constant R of $2{\cdot}8 \times 10^{-4}$.

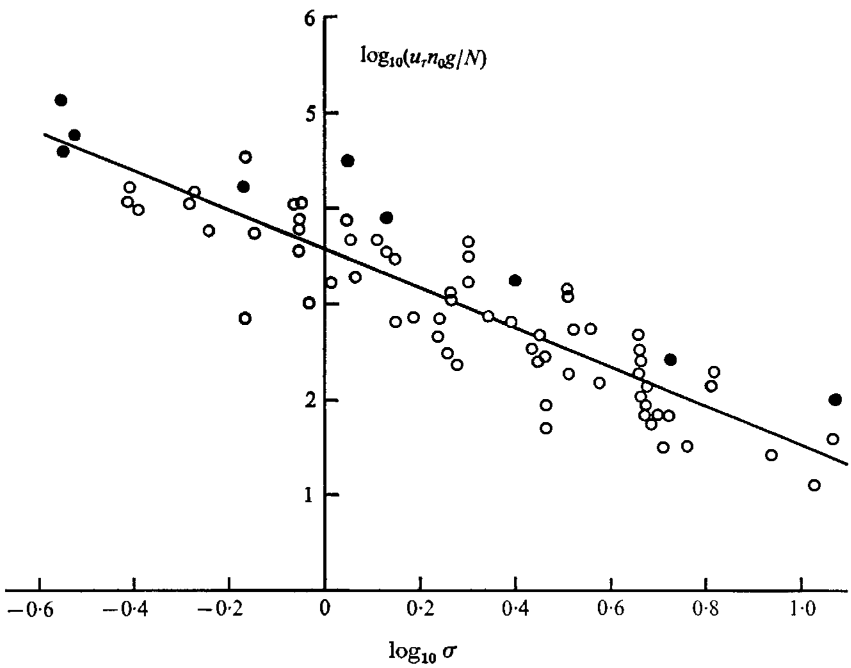

FIGURE 10. Measurements of particle deposition. ○, Friedlander & Johnstone; ●, Wells & Chamberlain. ——, $N/u_\tau n_0 = 2{\cdot}8\ 10^{-4}\ \sigma^2$.

One might now enquire what happens as the particle size is reduced and σ approaches zero. The answer must be that the boundary between the convection and diffusion regimes progressively approaches the edge of the viscous sublayer. Once it reaches that edge, it stays there, since the viscous sublayer may be regarded as a region predominantly of uniformly small eddies and characteristic times. Any further reduction in particle size cannot lead to a change in deposition rate due to convection. But when that stage is reached Brownian diffusion becomes significant: nonetheless, it is not difficult to make a combined account of the two mechanisms.

I shall not go into the details of such a calculation, but mention one rather common flow in which it is important: the inspired flow of dust-laden air into the human lung.

At moderate to vigorous levels of breathing the flow through the trachea, or wind-pipe, is turbulent, and one might imagine that dust particles present in the inspired air would be vigorously deposited on the trachea wall, there eventually to be rejected together with the drifting mucous layer. In consequence, one might regard the trachea as a kind of filter protecting the deeper parts of the lung from particle penetration.

Whilst this might be true of the larger particles, at any rate those that evade capture in the nose or mouth, it does not appear to be the case with the finer particles, less than about 1 μ in diameter. In fact, I understand that carcinoma of the lung is never found in the trachea, but always deeper down the respiratory tract.

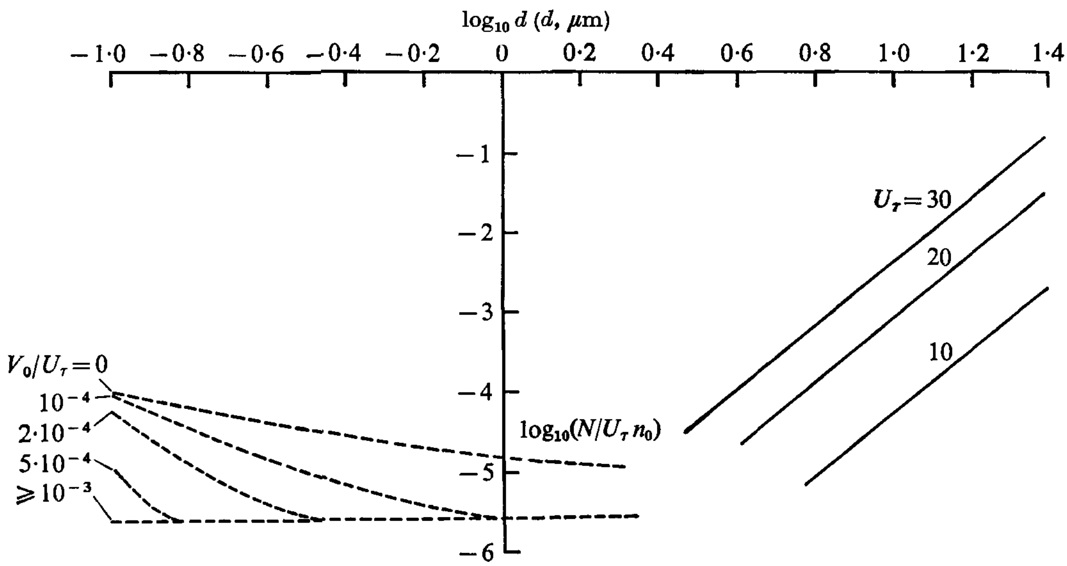

FIGURE 11. Estimated rates of particle deposition in the trachea. U_τ measured in cm s^{-1}.

One explanation, which I gave in a paper to the CIBA Foundation (1969) is that on account of a presumed temperature difference between the inspired airflow and the trachea wall, there is an evaporative flow away from the wall. For a temperature difference of the order of 1 °C, the evaporative flow velocity is very small: of the order of 10^{-2} mm s^{-1}. Yet it is not negligible compared with the velocity of migration of small particles in Brownian diffusion. The consequent effect on deposition rate is shown in figure 11. The appropriate part of the figure is that consisting of the broken curves. V_0 is the evaporative flow velocity which, for a temperature difference of, say, 2° C, has a value of about $5 \times 10^{-4}\ u_\tau$. The reduction in deposition rate compared with an isothermal flow is quite marked. It appears, then, that the effectiveness of the trachea as a fine particle filter is impaired by the evaporation from its wall and, during inhalation, especially through the mouth, those particles are free to descend more deeply into the bronchial tree.

8. The effect of particles on the flow in a jet ($d \sim 10^{-4}$ cm $- 10^{-2}$ cm)

The final aspect of pneumatic transport that I shall comment on briefly concerns the behaviour of particles in the 1 to 100 μ diameter range in a turbulent shear flow, notably that of a round air jet.

One can argue broadly that the effect of dust on such a jet is to add to its inertia, so that air entrained from its surroundings is less effective in slowing it down as it proceeds away from the orifice than it would be if the jet were unadulterated. Accordingly, we should expect the axial velocity to decay more slowly and the rate of spread to be smaller than in a pure air jet.

Such a conclusion is in agreement with what is observed experimentally, as can be seen from figure 12, based on the measurements of Laats (1966) but, on the theoretical side, it involves an inconsistency. It is this: if the only role of the particles is to add to the inertia or density of the gas, such particles must *travel* with the gas. But the gas slows down after it leaves the orifice; so too, then, must the particles. Since the only force available for slowing-down the particles arises from their motion relative to the gas, the initial assumption is violated.

In an attempt to resolve the dilemma, let us assume the particles to be fine, so fine that their relaxation time is small compared with the characteristic time of the turbulent energy-containing eddies, a condition satisfied by most of Laats' experiments. It implies that the particles are fully responsive to the turbulent fluctuations; but, when their concentration is not uniformly distributed across the flow, and that flow possesses a mean rate of strain, there exists an interaction between the radially migrating particles and the permanent mean rate of strain. If r is the radial co-ordinate, and U the mean axial velocity of the air, the axial component of a particle's velocity lags behind that of the fluid by the amount,

$$v' t^* \partial U / \partial r,$$

where v' is the radial component of the turbulent fluctuation. (In fact, in a jet, $\partial U / \partial r$ is negative.)

This velocity lag introduces a force on the particle and a corresponding reaction on the fluid. If the radial rate of mass migration of particles can be described in terms of an eddy diffusivity ν_m, it turns out that the force exerted on unit volume of the fluid in the direction of flow is

$$\nu_m (\partial \sigma_P / \partial r)(\partial U / \partial r).$$

(See note (vii).)

You will notice that the force is of the first order in the mass concentration. Although small for a dilute suspension, it is not negligibly so. Incidentally, you will also notice that, in form, the force bears a close similarity to what Prandtl predicted according to his mixing length theory for the turbulent transport of heat in a gas. That is hardly surprising for, in my view, particles genuinely exhibit a mixing length owing to the finite time they take to adjust to a change in environment. Where my argument differs from Prandtl's is in including, additionally, a transport due to eddy diffusivity, since the particles are assumed to respond fully to the turbulent part of the motion.

When the force is included in the equation of asymptotic motion of the gas in the jet, provided the mass concentration is small, that equation may be solved, as a series in the reciprocal of distance from the orifice or virtual origin.† Account can also be taken of the argument given previously concerning the diminution

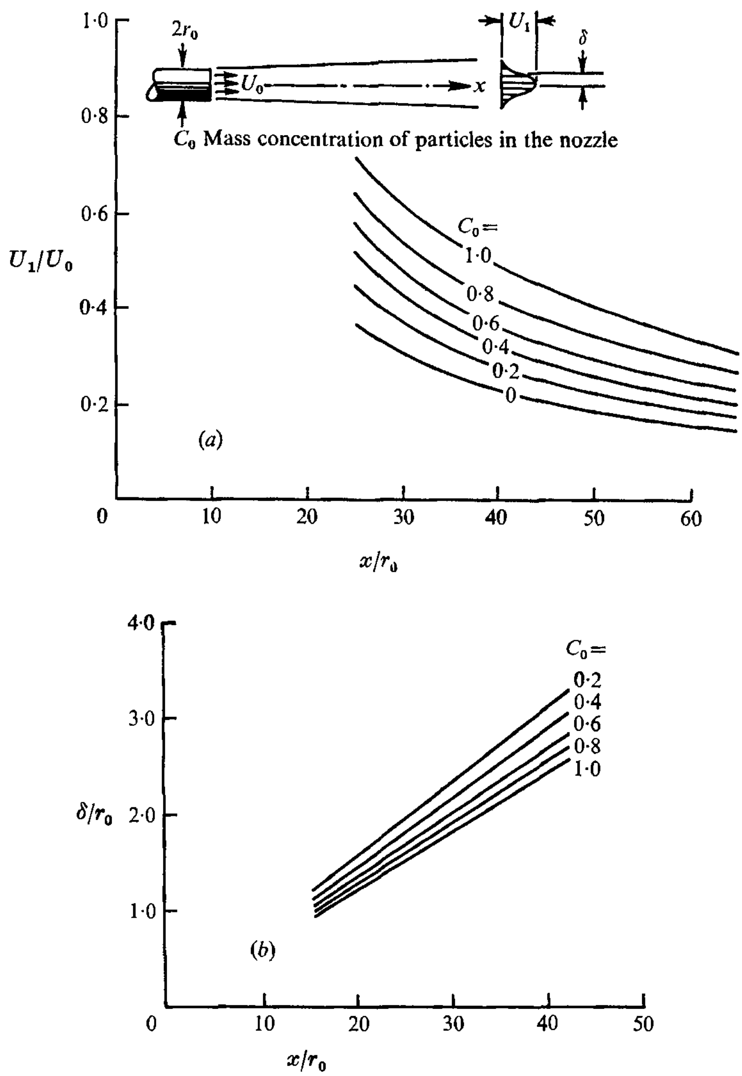

FIGURE 12. The observations by Laats of the velocity and radial growth of round, dust-laden, air jets: (a) velocity decay, (b) jet radius.

of eddy viscosity caused by the presence of the particles. I shall here omit the details, except to mention that the series contains a logarithmic term.

Having begun this talk in a gloomy key, later to be transposed by enthusiasm, it would be as well, in ending, to return to a mood of bewilderment: that series

† It is proposed to publish shortly a full description of this work.

solution of the asymptotic equation of jet motion is generated from a singular perturbation of the corresponding motion of a dust free jet. One of its coefficients is indeterminate!

Notes

(i) It may be argued that the ratio of the turbulent shear stress to the mean velocity gradient in the viscous sublayer on a smooth wall behaves like y^3, where y is measured from the surface. Interpreting that ratio as proportional to an eddy viscosity ν_T, consistency with the measurements of Laufer (1954) and Schubauer (1954), which approached the outer edge of the viscous sublayer, is achieved if

$$\nu_T/\nu \approx 10^{-3}(yu_\tau/\nu)^3.$$

Assuming equality between the eddy viscosity and the turbulent diffusivity of fine particles, the minimum value of the latter occurs at a distance of order d from the wall, appropriate to a particle in contact with the wall, and evidently is comparable with the Brownian diffusivity when

$$D/\nu \sim 10^{-3}(u_\tau d/\nu)^3. \tag{9}$$

(ii) The lift on a sphere translated with velocity W relative to a flow possessing a shear dU/dy is

$$L \sim \rho W d^3 dU/dy,$$

provided that $Wd/\nu \gg 1$. The Magnus force due to spin at a rate comparable with dU/dy is of similar order of magnitude.

In turbulent transport, W may be imagined to arise from a transverse movement of the sphere by an eddy through a distance l', the relaxation length, from a position where the mean gas velocity is U to another where it is $U+l'dU/dy$. If the characteristic velocity and time of the eddy are respectively u_τ and t_e, it follows that

$$W \sim u_\tau t^*(dU/dy)\{1+(t^*/t_e)^2\}^{-\frac{1}{2}},$$
$$\sim u_\tau t_e dU/dy,$$

if $t^* \gg t_e$, appropriate to massive particles occupying domains on the right of the diagrams in figure 2.

Outside the viscous sublayer, $dU/dy \sim u_\tau/y$ and $t_e \sim y/u_\tau$. It follows that

$$L/(mg) \sim (\rho/\rho_P)u_\tau^2/gy. \tag{N 1a}$$

For a much finer particle, such that $Wd/\nu \ll 1$, we may use Saffman's (1965) expression for the lift:

$$L \sim \mu W d(d^2\nu^{-1}dU/dy)^{\frac{1}{2}}.$$

If $t^* \ll t_e$, corresponding to the domains on the left of the diagrams in figure 2,

$$W \sim u_\tau t^* dU/dy.$$

Thus, outside the viscous sublayer,

$$L/(mg) \sim (d/y)(u_\tau^3/g\nu)(u_\tau y/\nu)^{-\frac{1}{2}}. \tag{N 2a}$$

Within the viscous sublayer, $dU/dy \sim u_\tau^2/\nu$. Then,

$$L/(mg) \sim (u_\tau^3/g\nu)(u_\tau d/\nu). \tag{N 3a}$$

Taking, for the present purpose, the viscous sublayer to occupy the region $0 < y < 10\nu/u_\tau$, it is evident that the lift/weight ratio given by (N 3*a*) exceeds that given by (N 2*a*) by at least $10^{\frac{3}{2}}$. In fact, it is only under the conditions implicit in (N 3*a*) that the lift due to shear in the gas can be appreciable, and then in rather extreme circumstances, brought about by a combination of high gas speed and comparatively large particle diameter, but not so large as to violate the condition $t^* \ll t_e$.

Departures from sphericity may produce additional lift forces. Thus, when $Wd/\nu \gg 1$, the lift ΔL due, in effect, to incidence is of order $\rho W^2 d^2$ and

$$\Delta L/(mg) \sim (\rho/\rho_P)(u_\tau^2/gd). \tag{N 1b}$$

When $Wd/\nu \ll 1$, an estimate of the extra lift may be guided by the analysis of the slow flow about spheriods (Happel & Brenner 1965). On such bodies, possessing a small eccentricity e and an incidence with respect to W, the lift is of order $\mu W d e^2$. Accordingly, outside the viscous sublayer

$$\Delta L/(mg) \sim e^2(u_\tau^2/gy), \tag{N 2b}$$

whereas, within the viscous sublayer,

$$\Delta L/(mg) \sim e^2(u_\tau^3/g\nu). \tag{N 3b}$$

(iii) Subsequent to an impact with the pipe wall at the point (0, 0), the equation of axial motion of a particle is

$$mU_s \partial U_s/\partial x = X,$$

where $$X = K_2(U-U_s)\{(U-U_s)^2+V_s^2\}^{\frac{1}{2}} \approx K_2(U-U_s)^2.$$

K_2 is the factor of proportionality between the drag on a particle and the square of its velocity relative to the gas.

The solution of the equation of motion satisfying $U_s(0) = 0$ is

$$U-U_s = m\{K_2 t + m/U\}^{-1}. \tag{12}$$

(iv) After one transverse flight, the particle either travels across the pipe and collides with the wall, rebounding with a radial velocity V_s, or remains in the body of the flow and acquires an axial velocity component approximately equal to that of the gas, until driven again to the wall by a turbulent eddy. In either case, the axial velocity defect, averaged over a large number of particles, is comparable with v_0 as given by (13).

(v) The suggestion that the transverse motion is initiated by a turbulent eddy might explain the change in behaviour of the solid phase that is observed to accompany a change in its concentration, Thus, as described in summary by Doig & Roper (1963), a high loading of solid material is found to be transported principally in the lower part of the pipe, under conditions approaching the minimum in the pressure-drop curves of figure 1, whereas a dilute system, at the same mean gas velocity, is dispersed across the pipe. Since, with an increase in particle concentration, the shear stress borne by the gas in the neighbourhood of the wall may be expected to decrease (for reasons given in §§ 2 and 5), hence (or otherwise, as discussed in § 6) leading to a reduction in the turbulent intensity there, the radial velocity V_s decreases and, with it, the radial distance over which the particles travel after an impact with the lower part of the wall.

(vi) A mass concentration σ_P of particles, each of mass m and carrying a charge q, induces a field at the radius r,

$$E = (q/m)(\epsilon_0 r)^{-1}\int_0^r \sigma_P r\,dr,$$

where ϵ_0 is the permittivity. If the particles are small enough for the drag to follow Stokes's law, they will tend to drift towards the wall under the action of the electrostatic field with the velocity

$$v^* = t^*(q/m)^2(\epsilon_0 r)^{-1}\int^r \sigma_P r\,dr,$$

and thus behave as if under the influence of an 'apparent gravity' g^*, such that

$$g^* = qE/m = v^*/t^*. \tag{20}$$

Near the wall, where it is supposed that the particles congregate, the rate of working by the turbulence against the electrostatic field is

$$-g^*\overline{v'\sigma'_P},$$

where v' and σ'_P are the fluctuations from the mean in velocity and concentration. But, if there is no net transport to or from the wall,

$$-\overline{v'\sigma'_P} = v^*\sigma_P,$$

where, for simplicity, we now treat the distribution of mean concentration as uniform with respect to radius. The rate of energy input to the turbulence from mean gas flow is

$$-\rho u_\tau^2 dU/dr,$$

in ratio to which the rate of working against the electrostatic field is

$$-g^*v^*\sigma_P/(\rho u_\tau^2 dU/dr).$$

However, on the assumption that the particles amass at such a distance from the wall that the eddy lifetime there is comparable with the relaxation time t^*,

$$-dU/dr \sim 1/t^*.$$

Hence, the above ratio of energy transfer rates becomes

$$(\sigma_P/\rho)(v^*/u_\tau)^2. \tag{22}$$

(vii) According to Stokes' law of resistance, the force on a single particle traversing the flow with velocity v' is

$$Kv't^*\,dU/\partial r;$$

and, if n' particles possess that velocity, the total force on unit volume of fluid is, on the average,

$$-K\overline{n'v'}t^*\partial U/\partial r.$$

Assuming an eddy diffusivity ν_m,

$$-\overline{n'v'} = \nu_m \partial n/\partial r,$$

from which it follows that the force on the fluid is

$$\nu_m(\partial\sigma_P/\partial r)(\partial U/\partial r),$$

since $Kt^* = m = \sigma_P/n$.

REFERENCES

BAGNOLD, R. A. 1941 *The Physics of Blown Sands and Desert Dunes.* London: Methuen.

CHEPIL, W. S. 1945 *Soil Sci.* **60**, 397.

CLARK, R. H., CHARLES, E. E., RICHARDSON, J. F. & NEWITT, D. M. 1952 *Trans. Instn Chem. Engrs*, **30**, 209.

CORRSIN, S. & LUMLEY, J. 1956 *Appl. Sci. Res.* **6** A, 114.

DALLA VALLE, J. M. 1948 *Micromeretics.* London: Pitman.

DOIG, I. D. & ROPER, G. H. 1963 *Aust. Chem. Engng.* Feb.

DUKLER, A. E., WICKS, M. & CLEVELAND, R. G. 1964 *A.I.Ch.E. J.* **10**, 38.

ELLISON, T. H. & TURNER, J. S. 1959 *J. Fluid Mech.* **6**, 423.

FARBAR, L. 1949 *Ind. Engng Chem.* **41**, 1184.

FRIEDLANDER, S. K. & JOHNSTONE, H. F. 1957 *Ind. Engng Chem.* **49**, 1151.

FUCHS, N. A. 1964 *The Mechanics of Aerosols.* Oxford: Pergamon.

GREEN, H. L. & LANE, W. R. 1964 *Particulate Clouds.* London: Spon.

HAPPEL, J. & BRENNER, H. 1965 *Low Reynolds Number Hydrodynamics.* Englewood Cliffs, N.J.: Prentice-Hall.

HINZE, J. O. 1959 *Turbulence.* New York: McGraw-Hill.

KLINE, S. J., REYNOLDS, W. C., SCHRAUB, F. A. & RUNSTADLER, P. W. 1967 *J. Fluid Mech.* **4**, 741.

KUNKEL, W. B. 1950 *J. Appl. Phys.* **21**, 820.

LAATS, M. K. 1966 *Inzh. fiz. Zh.* **10**, 1.

LAUFER, J. 1954 *NACA Tech. Rep.* 1174.

OWEN, P. R. 1960 *Aerodynamic Capture of Particles.* Oxford: Pergamon.

OWEN, P. R. 1964 *J. Fluid Mech.* **20**, 225.

OWEN, P. R. 1960 *Circulatory and Respiratory Mass Transport.* London: Churchill.

RICHARDSON, J. F. & MCLEMAN, M. 1960 *Trans. Instn Chem. Engrs*, **38**, 257.

SAFFMAN, P. G. 1965 *J. Fluid Mech.* **22**, 385. (Corrigendum; **31**, 3.)

SCHUBAUER, G. B. 1954 *J. Appl. Phys.* **25**, 188.

SEGLER, G. 1951 Pneumatic grain conveying. *Natn. Inst Agric. Engng Silsoe, Beds.*

SOO, S. L., TREZEK, G. J., DIMICK, R. C. & HOHNSTREITER, G. F. 1964 *Ind. Engng Chem.* **3**, 98.

SOO, S. L. 1965 *Proc. Symp. Single and Multi-component Flow Processes. Engng Res.* Pubn. 45, Rutgers Univ.

SOO, S. L. 1967 *Fluid Dynamics of Multiphase Systems.* Cambridge, Mass.: Blaisdell.

TCHEN, C. M. 1947 Mean value and correlation problems connected with the motion of small particles suspended in a turbulent fluid. Ph.D. Thesis, Delft.

THOMAS, D. G. 1963 *A.I.Ch.E.* **9**, 310.

TIEN, C. L. & QUAN, V. 1962 *A.S.M.E.* 62-HT-15.

Townsend, A. A. 1956 *The Structure of Turbulent Shear Flow*. Cambridge University Press.

Webster, C. A. G. 1962 *J. Fluid Mech.* **13**, 307.

Wells, A. C. & Chamberlain, A. C. 1967 *Br. J. Appl. Phys.* **18**, 1793.

Whitman, V. E. 1926 *Phys. Rev.* **28**, 1287.

Wood, S. A. & Bailey, A. J. 1939 *Proc. Instn Mech. Engrs*, **142**, 149.

Zingg, A. W. 1953 *Univ. Iowa Studies in Engng Bull.* no. 34.